African Invention of Mathematics

and the

Egyptian Way of Calculating the Circle

drafts, written for a conference and a journal, illustrated

African Invention of Mathematics

written for a conference, May 2014, worked over in June 2016

abstract

Today we have megabytes for saying little, whereas early scribes had small writing spaces for saying it all. We can't read their documents in the way we read a modern book. Consider for example problem number 32 in the Rhind Mathematical Papyrus, a famous Egyptian scroll from around 1 650 BC (copy of a lost scroll from around 1 850 BC). Ahmose or simply Ahmes divides 2 by 1 '3 '4 and obtains 1 '6 '12 '114 '228. What may appear as a long and tedious calculation with cumbersome unit fraction series reveals a fascinating theorem in form of a telling example when we play with the numbers. Imagine the inside of a granary in the shape of a right parallelepiped measuring 2 by 1 '3 '4 by 1 '6 '12 '114 '228 units. How long is the diagonal of the volume? Impossibly difficult to calculate? No, amazingly simple. The diagonal measures exactly

1 '2 '3 '4 plus 1 '6 '12 '114 '228 units

1 1 plus '3 '6 plus '4 '12 plus '114 '228 units

1 '2 '3 '76 units

Divide 2 by any number A and you obtain B

2 divided by A equals B

Let a right parallelepiped measure 2 by A by B units. How long is the diagonal of the volume? Exactly A plus B units.

Being a fan of experimental archaeology I applied the playing method to early and very early mathematics, the Rhind Mathematical Papyrus and further documents, among them the Lebombo bone from around 37 000 BP. Could the 29 tally marks on the baboon femur encode a calendar? Yes, they do: a lunisolar calendar. We'll find it by laying out pebbles in simple and pleasing patterns that require no numbers but might once have been encoded in a mythical story … Then follow the tally marks on the Ishango bone, 25,000 years old, interpreted in the same spirit – calendar lessons and geometric lessons for aspiring shamans in the African Stone Age.

(Mathematics, logic of building and maintaining, might well have begun with calendars that combine Earth and heavenly cycles and turn the world wherein we live into a metaphorical home, furthermore they provided a temporal structure for the organization of a society.)

Lebombo bone, Ishango bone, Blombos cave

A baboon femur from the Lebombo cave, eastern Congo, 37,000 years old, shows 29 tally marks that can be interpreted as a lunisolar calendar.

Gather a heap of small round pebbles of about the same size, each one representing a day (or a night). Lay out a line of as many pebbles as there are tally marks on the bone

```
   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
```

You may call this a 'female moon', as it corresponds to the female cycle. Add a slightly longer syncopic line, a 'male moon'

```
m  ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
f   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
```

Repeat the pattern, alternately a male and female moon, thus you get a lunar calendar

```
m  ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
f   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
m  ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
f   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
m  ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
f   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
m  ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
```

Now for a solar calendar. Begin with a female moon, the same number of pebbles as there are tally marks on the bone

```
   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
```

Find and remove the pebble in the middle

```
   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ - ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
```

Make squares of the shorter lines

```
   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
```

Connect them with a pebble in the top line and another pebble in the bottom line

```
○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
```

Remove the top line and you have a regular year that will require an occasional leap day

```
○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○   ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
```

Once an aspiring shaman or shamaness laid out the above sequence of calendar patterns, instructed by an experienced shaman or shamaness, he or she could have repeated them anywhere anytime from a copy of the bone, *without using any number*, but maybe following a mythical story encoding this calendar.

Now there is a lunisolar aspect (here given in numbers for the sake of simplicity): 19 lines of 14 pebbles are 266 days and correspond to 9 lunations or synodic months counted in the 30 29 30 mode. 21 periods of 14 days plus the additional day represented by the pebble in the center of the bottom line are 295 days and correspond to 10 lunations.

Ishango bone, African math lessons from the Stone Age

Aspiring young shamans may have learned how to calculate calendars by solving problems encoded in the tally marks of bones like the Ishango bone from Zaire, 25 000 BP (numbers of tally marks according to Ian Stewart, Taming the Infinite, Quercus London 2008/9; picture the marks in vertical position, and longer).

SIDE A, calendar lessons – left edge, downward 11 21 19 9 marks

/ / / / / / / / / / /   /   / / / / / / / / / / / / / / / / / / /   / / / / / / / / /

Remember the 'female moon' of 29 pebbles and the 'male moon' of 30 pebbles. Find male moons in the above tally marks.

11 19 sum 30   21 9 sum 30   11 21 19 9 sum 30 30

Lay out a field of 11 by 21 pebbles. Remove 19 by 9 pebbles. How many pebbles remain?

11 x 21 minus 19 x 9 remain 30 30

Lay out a field of 21 by 19 pebbles. Remove 11 by 9 pebbles. How many pebbles remain?

21 x 19 m 11 x 9 r 30 30 30 30 30 30 30 30 30 30

SIDE A, more calendar lessons – right edge, upward 19 17 13 11 marks

/ / / / / / / / / / / / / / / / / / /   / / / / / / / / / / / / / / / / /   / / / / / / / / / / / / /   / / / / / / / / / / /

19 11 sum 30   17 13 sum 30   19 17 13 11 sum 30 30

19 x 13 minus 17 x 11 remain 30 30

19 x 17 minus 13 x 11 remain 30 30 30 30 30 30

SIDE B, a geometrical formula – lower half, upward 7 5 5 10 marks

/ / / / / / /   / / / / /   / / / / /   / / / / / / / / / /

Let a square measure 5 by 5 paces. How long is the diagonal? 7 paces. And if the side measures 7 paces? 10 paces.

If a square measures 5 by 5 paces
or multiples thereof
the diagonal measures 7 paces
or a multiple thereof

And if the side measures 7 paces
or a multiple thereof
the diagonal measures 5 5 sum 10 paces
or multiples thereof

SIDE B, another geometrical formula – upper half, upward 8 4 6 3 marks

/ / / / / / / /   / / / /   / / / / / /   / / /

Let a double square measure 8 by 4 paces. How long is the diagonal? 6 plus 3 paces.

double square 8 by 4 paces (or multiples)   diagonal 6 plus 3 paces (or multiples)

The oldest geometrical drawing known so far is a row of rhombs engraved on an ocher prism found in the Blombos Cave, South Africa, Middle Stone Age, 75 000 BP, according to my hypothesis rendering the act of creation in flaps of the wings of Blue Crane, perhaps combined with a calendar, maybe of seven days? 38 weeks of seven days are again 9 lunations or synodic months, 30 29 30 29 30 29 30 29 340 sum 266 days. (We can hope for more answers to these questions from new archaeological findings.)

How to use the Ishango formula of the square

The group of tally marks on the lower half of side B can be read as formula of the square

///////   ///// /////   //////////   7 5 5 10

if the side of a square measures 5 paces
the diagonal measures 7 paces
and if the side measures 7 paces
the diagonal measures 5 5 or 10 paces

Let us correlate multiples of 5-7 and of 7-10 and calculate squares by using number pairs from both lines

5-7 10-14 15-21 20-28 25-35 30-42 35-49 ...

7-10 14-20 21-30 28-40 35-50 42-60 49-70 ...

side 27 or 20 plus 7   20-28 plus 7-10   diagonal 28 plus 10 or 38 (38.183...)

side 63 or 35 plus 28   35-49 plus 28-40   diagonal 49 plus 40 or 89 (89.085)

side 70 or 35 plus 35   35-49 plus 35-50   diagonal 35 plus 35 or 99 (98.994...)

side 112 42-60 20-42 20-28 20-28 diagonal 158
side 112 49-70 35-49 28-40 diagonal 159
side 112 diagonal between 158 and 159 (158.391...)

The Ishango formula for the calculation of the square – 7 5 5 10 – yields fine results.

Piling Bricks and Blocks and Numbers

Egyptian way of calculating the circle, written for a journal in May/June 2016

abstract

The author summarized a good half century of private studies in the triangle of language whose corners are

life with needs and wishes

mathematics, logic of building and maintaining, based on the formula $a = a$

art, human measure in a technical world, based on Goethe's formula
all is equal, all unequal …, known to artists of all times

Galileo famously wrote that the book of nature is written in the language of mathematics … God may understand all of nature in mathematical terms while for us logic falls apart into equal unequal (the infinite being the border between them, equal unequal in itself). We have to consider the other side of logic as well. Consequently there are two criteria for the height of a civilization: its mathematical or technological height, and its cultural height – how well is technology integrated into life and nature? A technologically simple culture can thus be higher on the cultural or sociological level. Formula for understanding early civilization: simple yet complex.

A stable civilization requires a balance of the above realms or aspects, life and mathematics and art. Meaning that a Stone Age culture could not have sent a missile on the moon, but also that a marvel of early civilization like the Egyptian pyramids are unimaginable without a solid body of mathematics and geometry. The following paper shows an array of early methods based on addition, reconstructed from geometry underlying Renaissance art, Stone Age patterns, Egyptian pyramids, and the Rhind Mathematical Papyrus. The pre-Greek methods require patterns that get clumsy when rendered in the formalism of our time, so they are given in their proper form.

Teaching mathematics would better begin with the beginning: with early methods giving the pupil an idea of what mathematics is and can achieve. The author gave free tutorials for two care organizations and began with a Stone Age 'computer', the Göbekli Tepe calendar as of 12,000 years ago, calculating the diagonal of the square in the Egyptian manner, and further number games. Once a pupil reached a basic understanding, going on with actual school mathematics became easier.

Mathematical techniques mirror the technology of a given era. Analysis goes along with mechanics, while Ancient Egypt, known for breathtaking buildings consisting of millions and millions of bricks or stone blocks, relied on simple yet clever additive methods – piling of bricks and blocks mirrored in the adding of numbers …

A simple formula might have defined the square for several ten thousand years. If the side of a square measures 5 paces or multiples thereof, the diagonal measures 7 paces or multiples thereof, and if the side of a square measures 7 paces or multiples thereof, the diagonal measures 2 x 5 = 10 paces or multiples thereof. Later on some early mathematicians wondered what happened if the side measures 5 + 7 = 12 paces. Does the diagonal measure 7 + 10 = 17 paces? Yes, it does. And then you can go on this way and establish what I call a number column the algorithm of which is quite easily found

```
    1       1       2
        2       3       4
            5       7      10
               12      17      24
                   29      41      58
                       70      99     140
                          ...     ...     ...
```

If the side of a square measures 70 units, the diagonal measures practically 99 units. By the way, this number column has an exact equivalent in the continued fraction for the square root of 2. Now use the same algorithm for calculating the double square, only that the factor is 5, and you can repeatedly half the numbers of a line

```
1       1       5
    2       6      10
    1       3       5
        4       8      20
        2       4      10
        1       2       5
            3       7      15
               10      22      50
                5      11      25
                   16      36      80
                    8      18      40
                    4       9      20
                       13      29      65
                           42      94     210
                           21      47     105
                               68     152     340
                               34      76     170
                               17      38      85
                                   55     123     275
                                      178     398     890
                                       89     199     445
                                          288     644    1440
                                          144     322     720
                                           72     161     360
```

Can you find the so-called Fibonacci sequence (below) and the so-called Lucas sequence (above) in the number column? The quotient (L/F) approximates the square root of 5

```
L   1  3  4  7  11  18  29  47  76  123  199  322  …
F   1  1  2  3   5   8  13  21  34   55   89  144  …
```

Jean-Philippe Lauer discovered the Sacred Triangle 3-4-5 in the form of 15-20-25 royal cubits in the so-called King's Chamber of the Great Pyramid: diagonal of the short wall 15 rc, length of the chamber 20 rc, diagonal of the volume 25 rc. Now this triangle allows a systematic calculation of the circle. Begin with a grid of 10 by 10 royal cubits or 70 by 70 palms or 280 by 280 fingerbreadths or simply fingers. The ends of the horizontal axis and of the vertical axis define 4 points of the circumference of the inscribed circle while 8 more points are given by the Sacred Triangle 3-4-5

```
. . . . . d . . . . .
. . e . . . . . c . .
. f . . . . . . . b .
. . . . . . . . . . .
. . . . . . . . . . .
g . . . . + . . . . a
. . . . . . . . . . .
. . . . . . . . . . .
. h . . . . . . . l .
. . i . . . . . k . .
. . . . . j . . . . .
```

The circle a b c d e f g h i j k l a has eight longer arcs measuring practically 90 fingers each, and four shorter ones measuring practically 40 fingers each, yielding in all 880 fingers or 220 palms. Divide this number by the diameter 280 fingers or 70 palms and you get 22/7 for pi.

Now let us consider the polygon a b c d e f g h i j k l a and calculate the periphery. The shorter side is given by the square root of 2, and the longer side by the square root of 10, or the square root of 2 times the square root of 5. For the periphery we get

```
4 x sqrt2 royal cubits  plus  8 x sqrt2 x sqrt5 royal cubits
```

Now the arcs of the circle are slightly longer than the sides of the polygon. We can compensate for this by choosing values from the number column that are slightly bigger than the actual square roots: 10/7 for the square root of 2, and 9/4 for the one of 5

```
4 x 10/7 royal cubits  plus  8 x 10/7 x 9/4 royal cubits

4 times 10 palms  plus  8 times 10 times 9/4 palms

4 times 10 palms  plus  2 times 10 times 9 palms

40 palms plus 180 palms = 220 palms
```

Divide the 220 palms of the rounded periphery by the 70 palms of the diameter and you obtain again 22/7 for pi. So far so good. Now let us consider the finer grid 50 by 50. The inscribed circle passes the four ends of the horizontal and vertical axes, plus four points defined by the Sacred Triangle 3-4-5 in the form of 15-20-25, plus four more points defined by the triple 7-24-25. The polygon has 20 sides. If you calculate the rounded periphery with 17/12 for the square root of 2 and again with 9/4 for the square root of 5 you obtain the circumference 157 units, and if you divide this measure by the diameter 50 units you get 157/50 = 3.14 for pi.

Let us proceed from the initial grid 10 by 10 to a second grid 50 by 50 to a third grid 250 by 250 to a fourth grid 1250 by 1250, and so on. The radius of the inscribed circle measures 5 25 125 625 … units, while 8 plus 8 plus 8 plus 8 … points of the polygon and circumference are defined by the following triples

```
3-4-5   15-20-25   75-100-125   375-500-625   ...
         7-24-25   35-120-125   175-600-625   ...
                   44-117-125   220-585-625   ...
                                336-527-625   ...
                                              ...
```

If you know a triple a-b-c and wish to find the next one calculate these terms:

plus minus 4b plus minus 3a   plus minus 3b plus minus 4a   5c

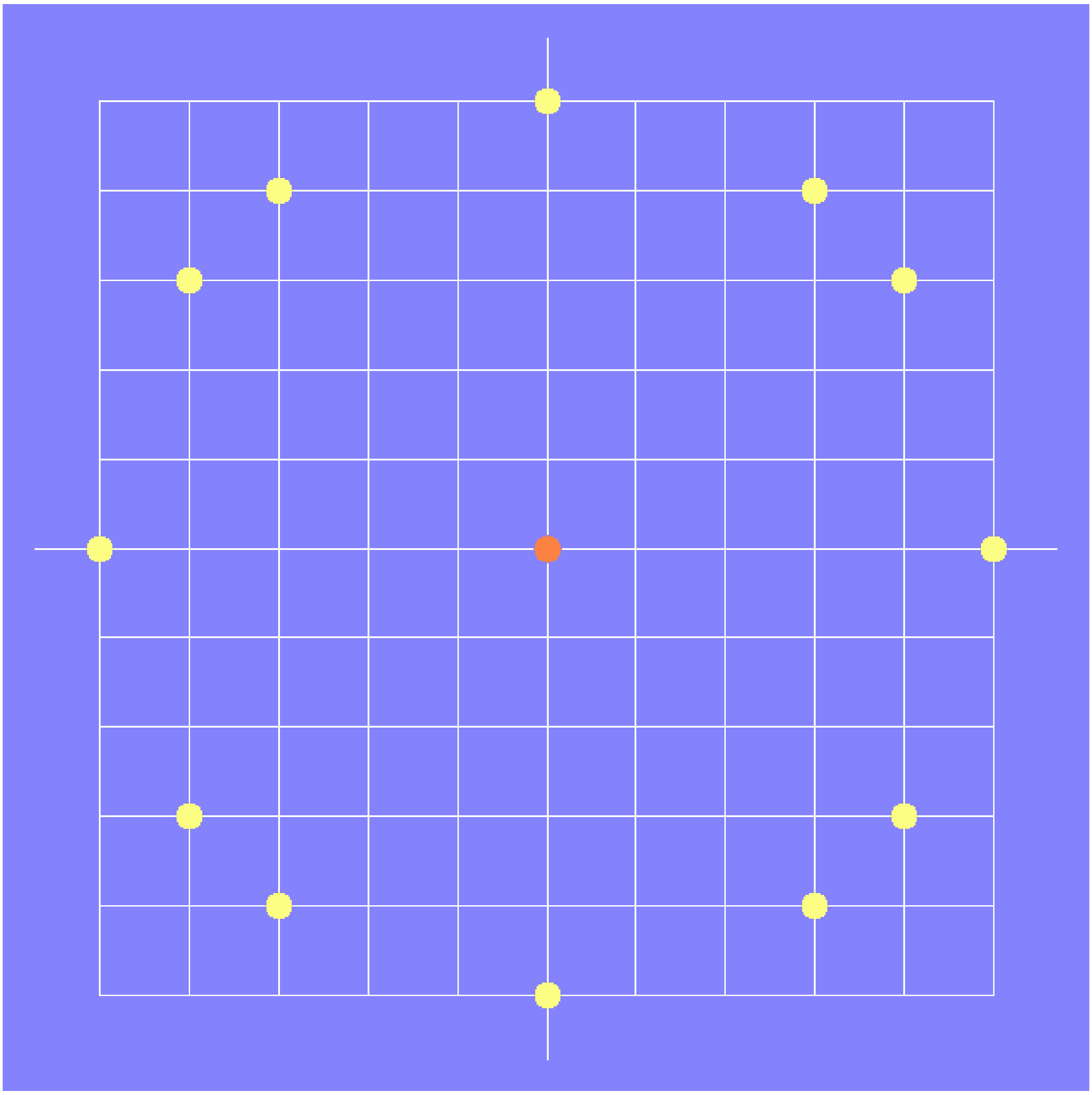
The first terms provide four results each. Use the positive numbers that end neither on zero nor five. By connecting the 12, 20, 28, 36 ... points of the grid you will obtain a sequence of rounder and rounder polygons that slowly approximate the circle. Their side lengths are whole number multiples of the square roots of 2 or 5 or 2 x 5, and these roots can be approximated by the above number columns. In other words: we have a systematic method for calculating the circle …

Remember the first pi value 22/7 and the second one 157/50. They can be found via two additive number sequences of another simple yet clever algorithm

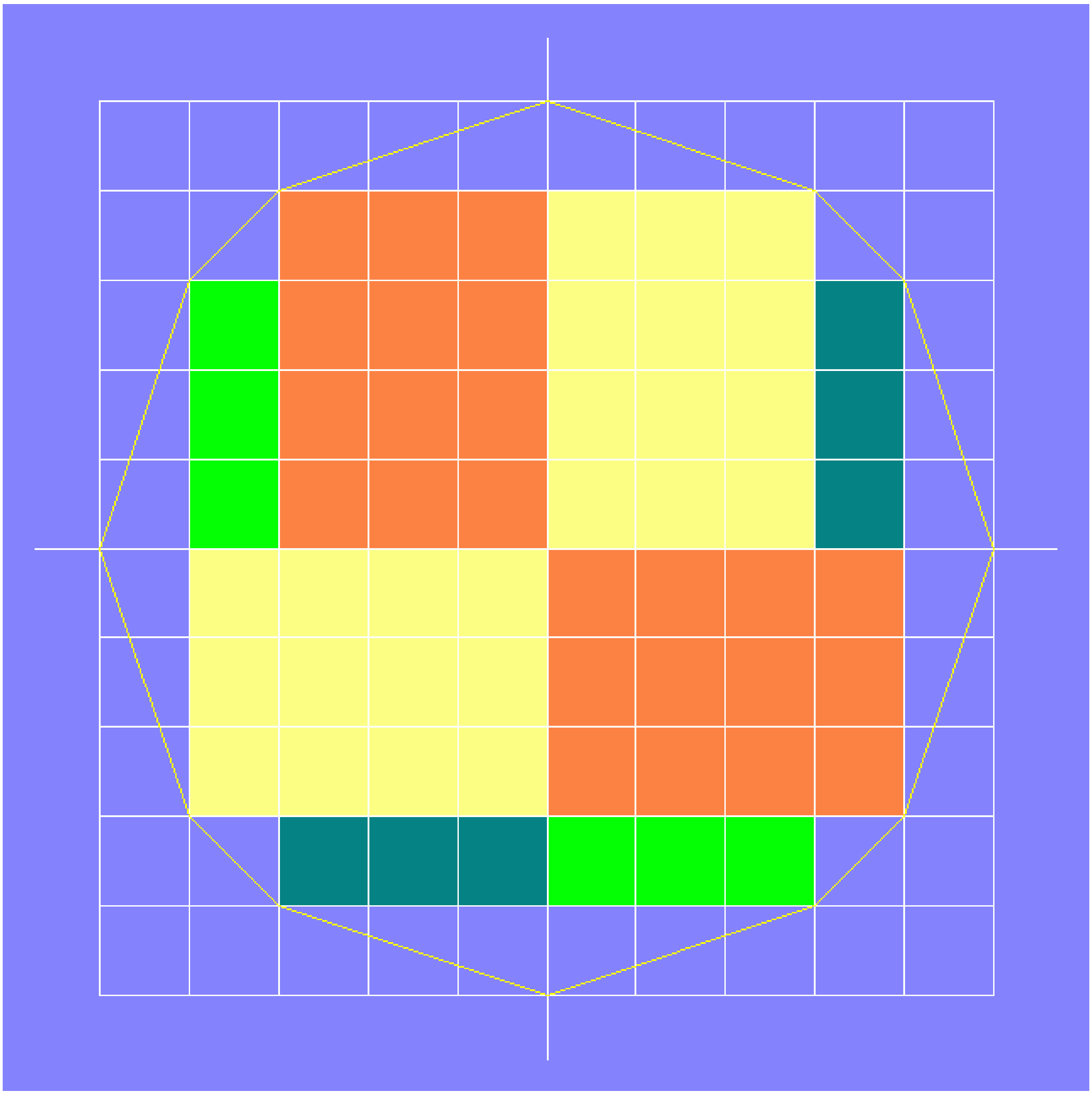
```
4/1  (plus 3/1)  7/2  10/3  13/4  16/5  19/6  22/7  25/8  28/9

3/1  (plus 22/7)  25/8  47/15  …  157/50  …  311/99  …  377/120
```

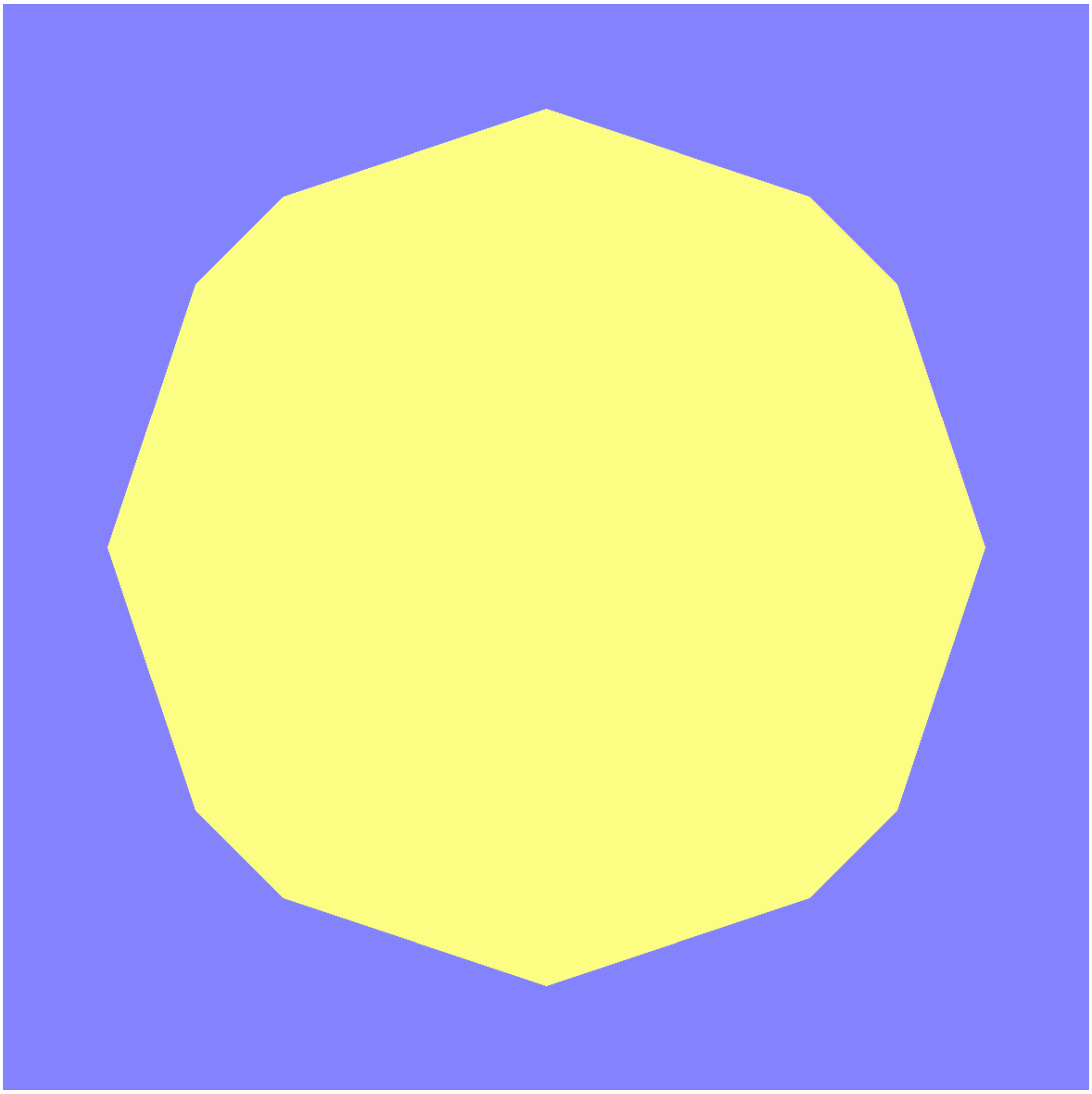
Many values instead of a single one have the advantage that you can choose the value that comes handy in a given calculation. The side of the initial square measures 10 royal cubits or 70 palms. How long is the diagonal? Consulting the first number column we find 99 palms. Now draw a circle around the square. The diameter equals the diagonal of the square and measures 99 palms. How long is the circumference of the circle? Consult the second pi sequence and you find 311 palms. What is the exact length of the circle circumscribing the square 10 by 10 royal cubits? 311.00180… royal cubits. You can operate with integers, the mistakes even out amazingly well.



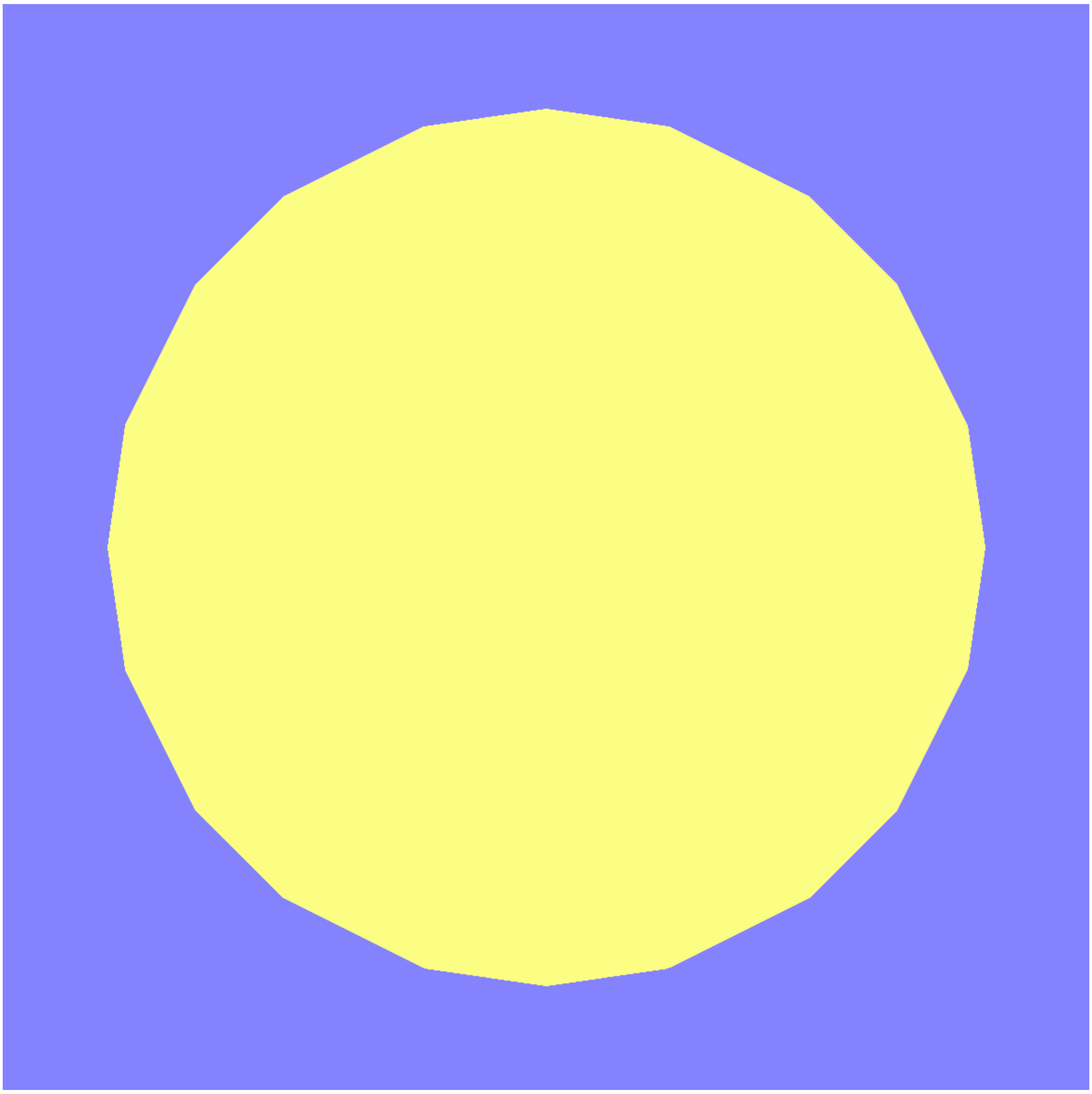
key figure, grid 10 by 10, Sacred Triangle 3-4-5



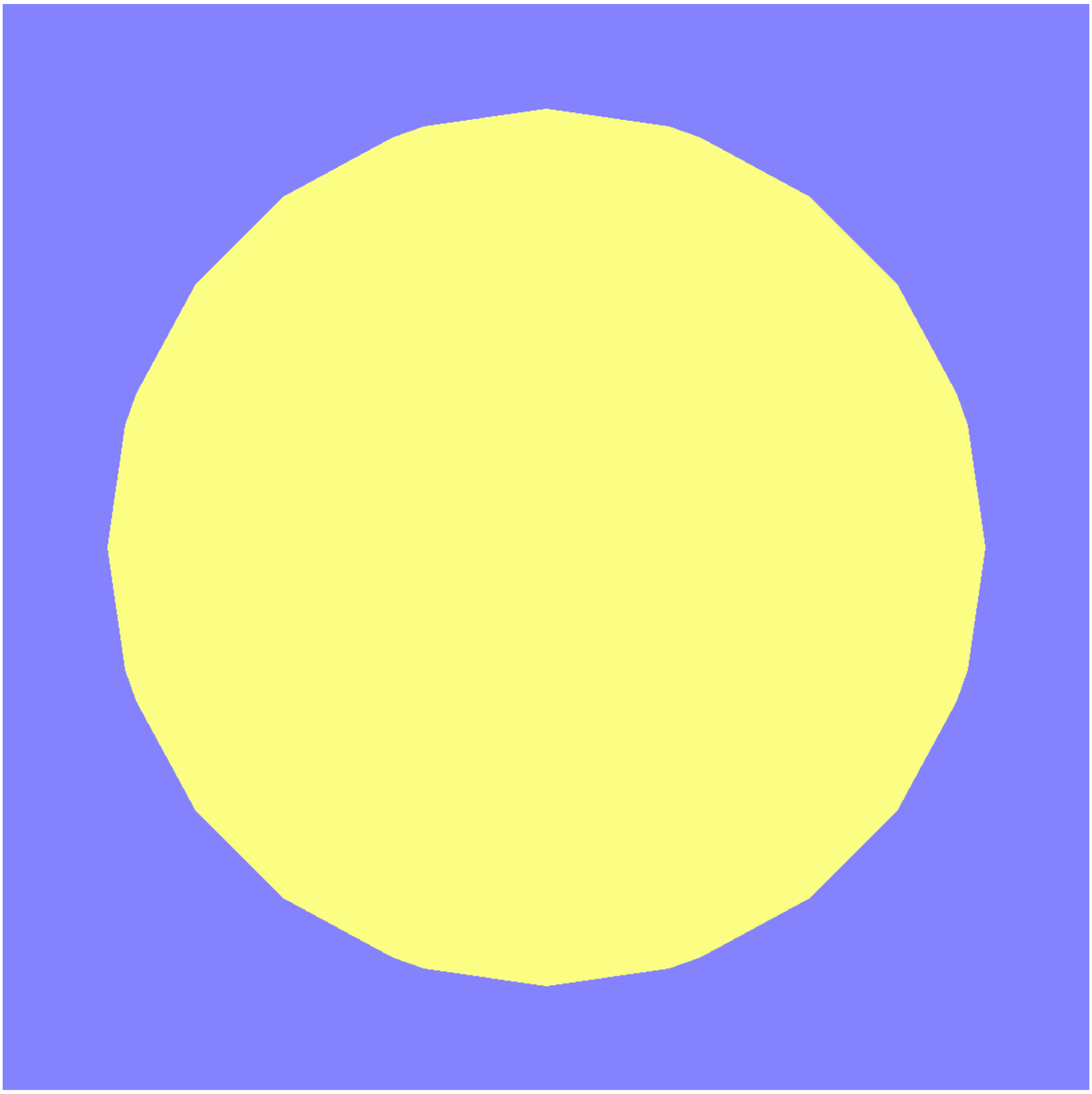
variation of the key figure



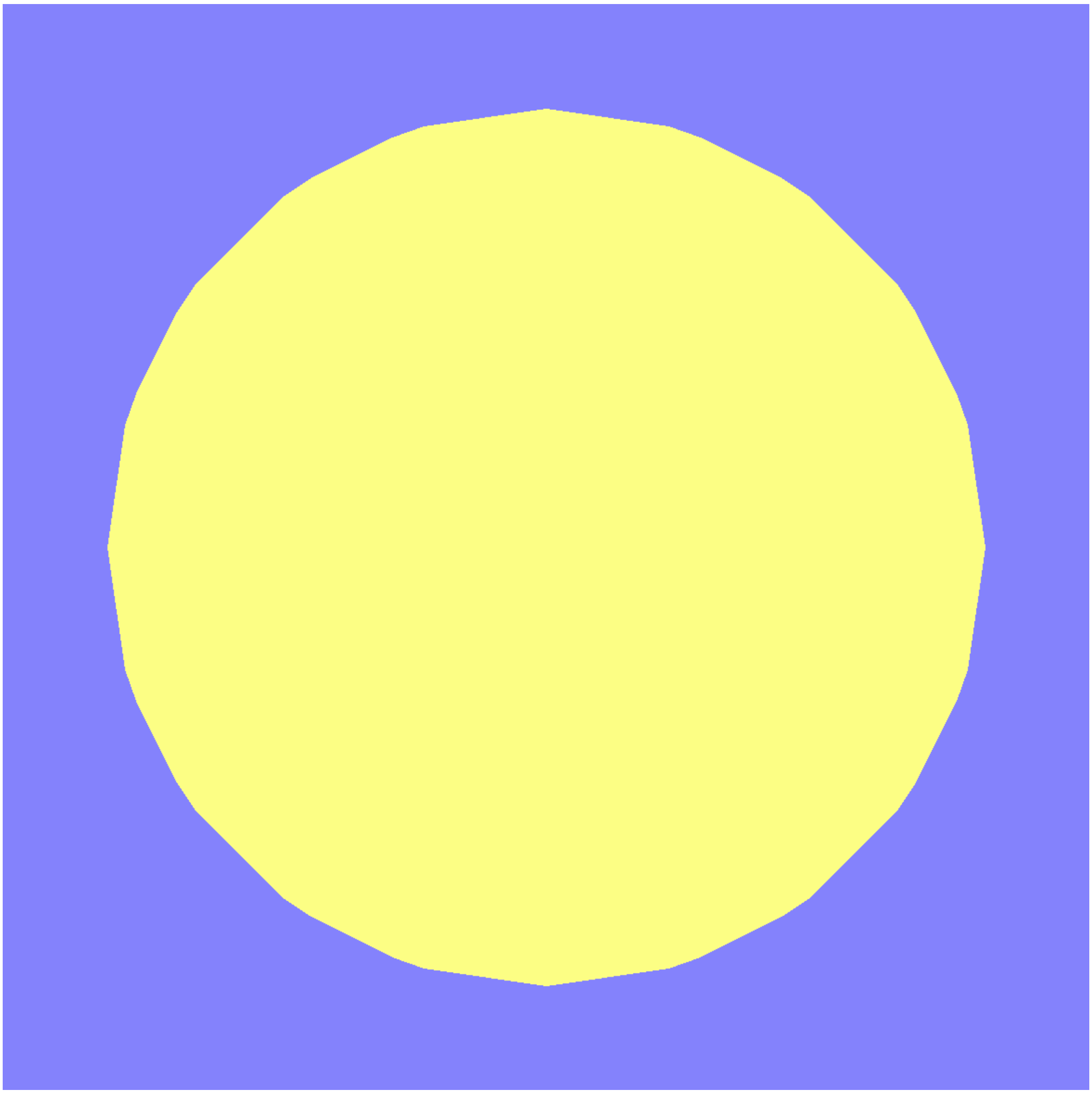
polygon based on the Sacred Triangle 3-4-5



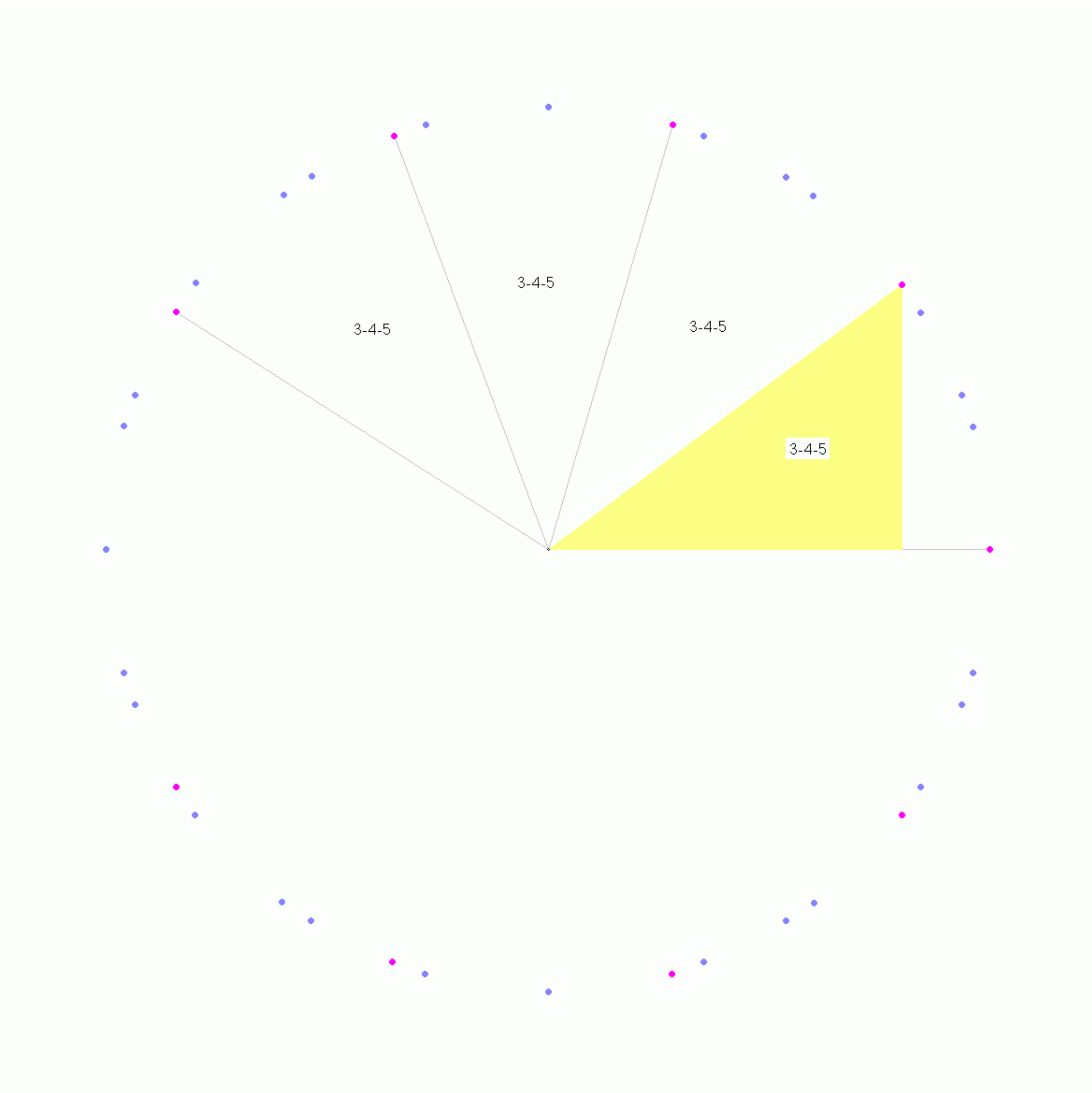
polygon based on the triples 3-4-5 and 7-24-25



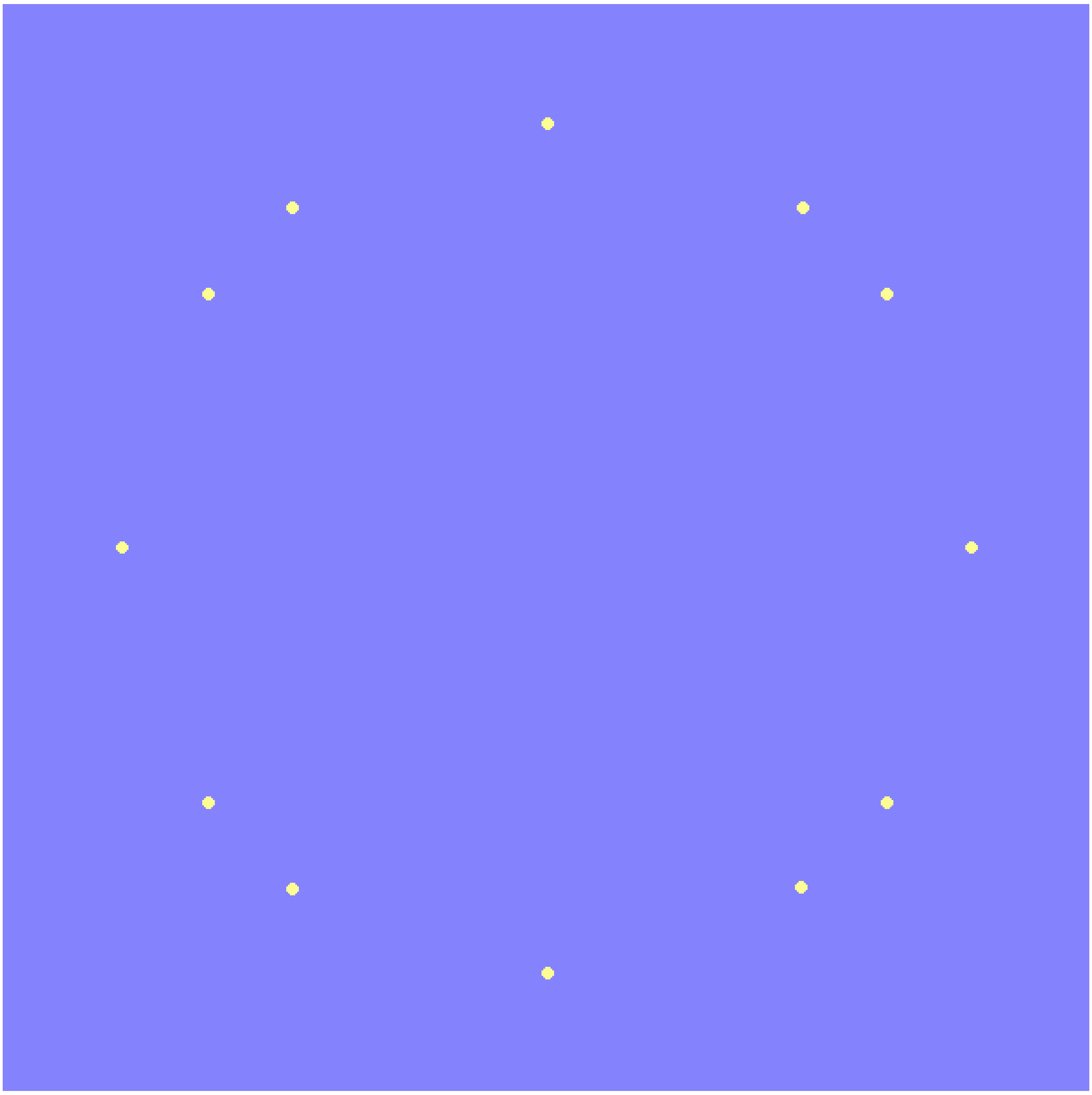
polygon based on the triples 3-4-5 and 7-24-25 and 44-117-125



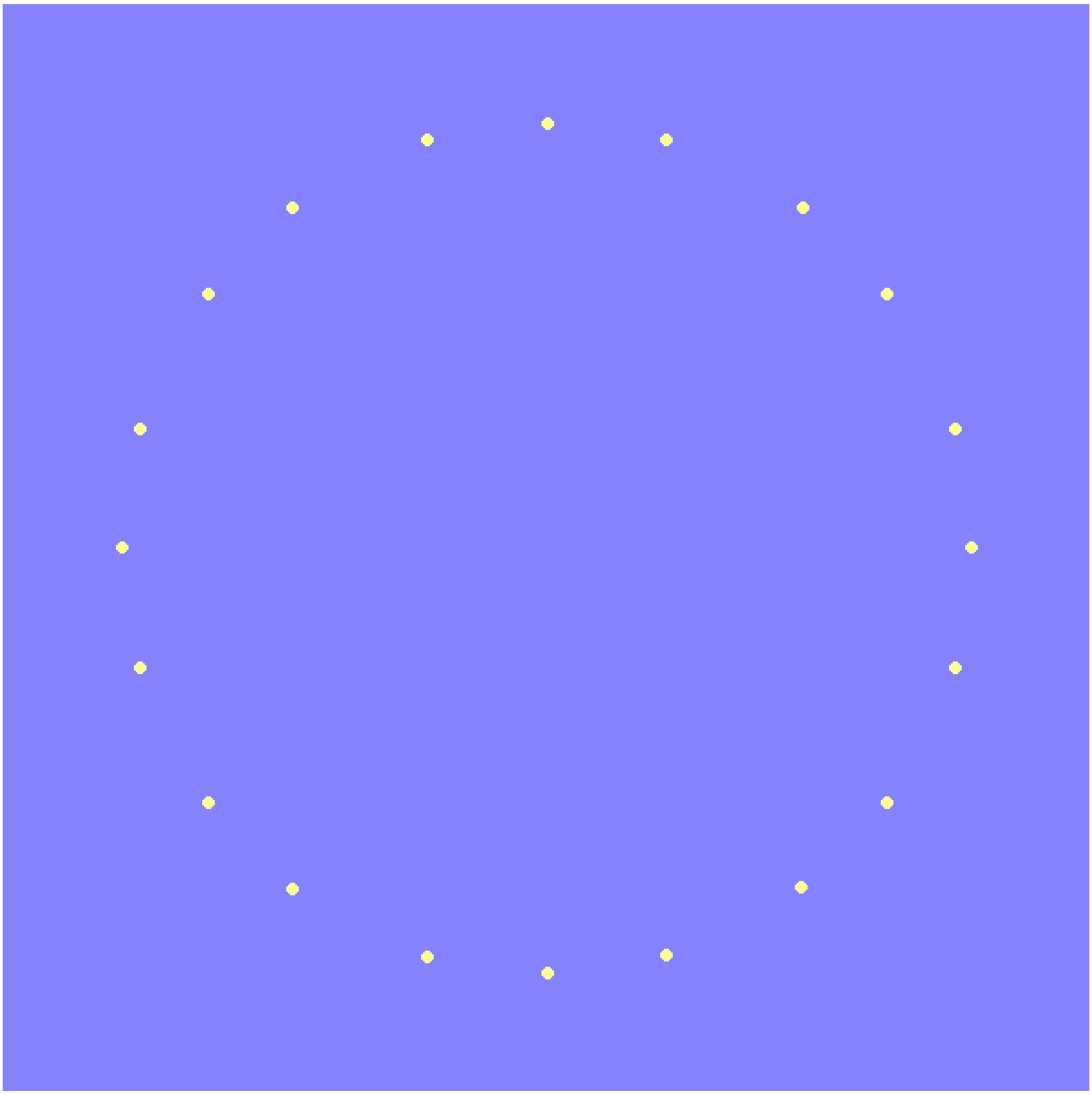
polygon based on the triples 3-4-5 and 7-24-25 and 44-117-125
and 336-527-625



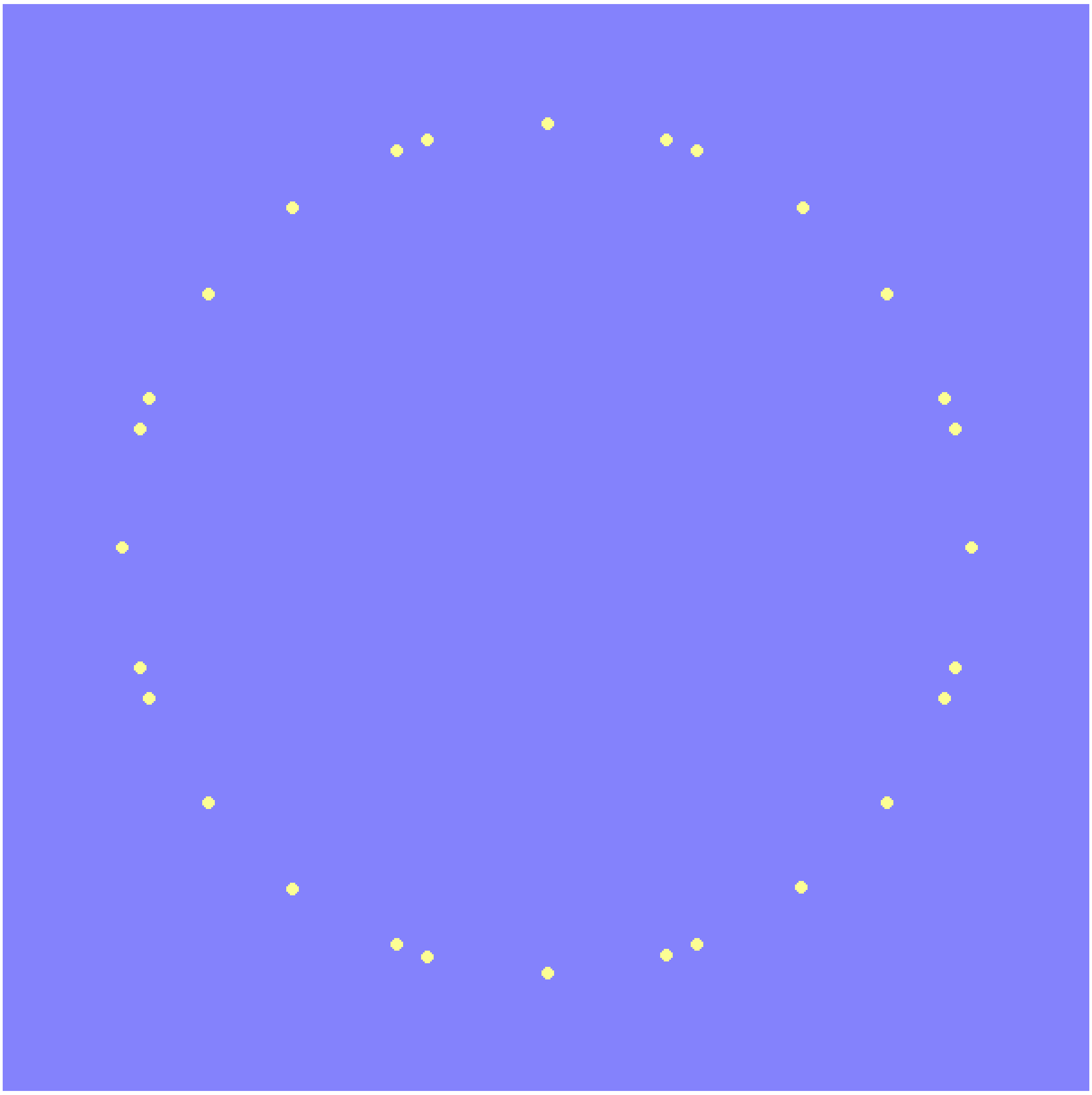
points of the polygon generated by a rotating radius
and the small angle of the Sacred Triangle 3-4-5



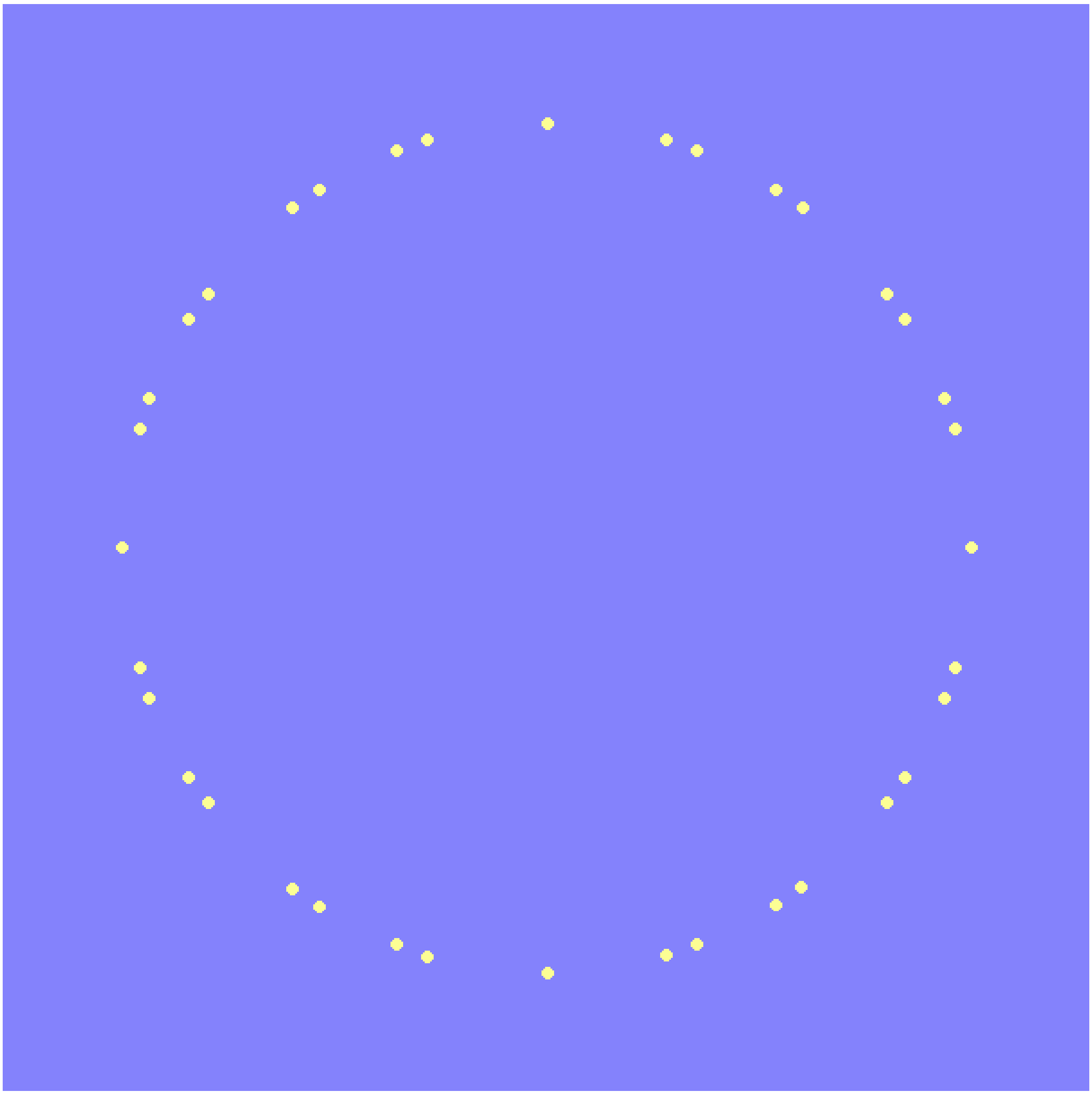
first 12 points of the polygon and circle



first 20 points of the polygon and circle



first 28 points of the polygon and circle



first 36 points of the polygon and circle

References

Jean-Philippe Lauer's discovery of the 'Sacred Triangle' 3-4-5 in the form of 15-20-25 royal cubits is mentioned in various books on the Egyptian pyramids. The Ishango bone is depicted for example in the book by Ian Stewart mentioned in the running text, here again: Taming the Infinite, Quercus London 2008/9. All other discoveries or hypotheses are mine, beginning with the number columns for the calculation of the square, from 1979, when studying the lunettes above Leonardo da Vinci's mural in the former refectory of the monastery Santa Maria delle Grazie at Milan. Analogous number columns for the cube and equilateral triangle and hexagon (square root of 3), for the double square (square root of 5), and for the doubling of a volume (cube root of 2) followed in the autumn of 1993, and the systematic method of calculating the circle in February 1994. See various pages on my website www.seshat.ch